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THEORETICAL MODELLING OF AN ABLATION PLASMA DYNAMICS

BY

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Abstract. Using the Fractal Theory of Motion in the form of Scale Relativity Theory with arbitrary constant fractal dimension, various behaviors of an ablation plasma are analyzed. More precisely, we show that in the expansion process of an ablation plasma three distinct "moments" are emphasized: the "Coulomb moment", the "thermal moment" and the "cluster moment".

Keywords: fractal; ablation plasma; non-differentiability.

1. From Differentiability to Non-Differentiability in the Dynamics of an Ablation Plasma

The ablation plasma (plasma produced by the laser-material interaction) can be assimilated to a complex system, taking into account both its structure and its functionality.

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The classical models used in the description of the dynamics of such a plasma are based on the hypothesis of the differentiability of the dynamic variables that can characterize it (Badii and Politi, 1997; Michell, 2009).

These models only partially describe the dynamics of ablation plasmas; the instabilities of the dynamics of these structures, however, involve non-differentiable procedures. We will use non-differentiable mathematical models (Nottale, 2011; Mercheş and Agop, 2016) to describe dynamics in ablation plasmas. More precisely, in the following we will analyze various “dynamics” of the ablation plasma using the Fractal Theory of Motion in the form of Scale Relativity Theory in arbitrary and constant fractal dimension (Mercheş and Agop, 2016).

2. Nonlinear Behaviors in the Expansion of an Ablation Plasma

The process of expansion of the ablation plasma contains in its evolution three “moments” and, implicitly, three more important components: first “the Coulomb moment”, then “the thermal moment” and, finally, the “cluster moment”. So, according to the superposition principle of scale resolutions, we must operate simultaneously with three scale resolutions, namely Coulomb scale resolution, thermal scale resolution and cluster scale resolution. In such a context, any global variable that will describe such dynamics will be the expression of the sum of three equivalent variables, one of Coulomb type, one of thermal type and other of cluster type. Next, let's “make explicit” each of these “moments”.

i) Behaviors at Coulomb scale resolution

The separation of electrical charges due to the interaction of laser radiation with matter induces a local electric field, whose equation of evolution is of the form (Jackson, 1999):

$$\mu\varepsilon\frac{\partial^2\vec{e}}{\partial t^2}+\mu\frac{\partial\vec{j}_c}{\partial t}=\Delta\vec{e}-\nabla(\nabla\cdot\vec{e}) \tag{1}$$

or, using Ohm's law for local current density,

$$\vec{j}_c=\sigma\cdot\vec{e} \tag{2}$$

the equation:

$$\frac{\mu\varepsilon}{\sigma}\frac{\partial^2\vec{j}_c}{\partial t^2}+\mu\frac{\partial\vec{j}_c}{\partial t}=\frac{1}{\sigma}\left[\Delta\vec{j}_c-\nabla\left(\nabla\cdot\vec{j}_c\right)\right] \tag{3}$$

In previous relationships, the ablation plasma is characterized by electrical permittivity ε, magnetic permeability μ and electrical conductivity σ.

"Δ" is Laplace's operator, "∇" is the gradient operator, and "∇" is the divergence operator.

Considering now that in Eq. (3) the method of separating variables can be applied, in the form:

$$\vec{j}_c(\vec{r},t)=\vec{R}_c(\vec{r})T_c(t) \tag{4}$$

it is transformed as follows:

$$\frac{1}{\sigma}\left[\Delta\vec{R}_c-\nabla\left(\nabla\cdot\vec{R}_c\right)\right]+\Lambda\vec{R}_c=0 \tag{5}$$

$$\frac{\mu\varepsilon}{\sigma}\frac{d^2T_c}{dt^2}+\mu\frac{dT_c}{\partial t}+\Lambda T_c=0 \tag{6}$$

where $\Lambda = const. > 0$ is the constant of separation of variables.
Solution of the Eq. (6),

$$T_c = T_0\, exp(-\delta t)\sin(\Omega t+\varphi) \tag{7}$$

where,

$$\frac{\sigma}{\varepsilon}=2\delta,\quad \Omega=(\Omega_0^2-\delta^2)^{1/2},\ \ \Omega_0^2=\frac{\Lambda\sigma}{\mu\varepsilon} \tag{8}$$

$$T_0 = const.,\ \ \varphi = const.,\qquad \Omega_0>\delta$$

specifies that the current density of the ablation plasma at the Coulomb scale resolution (Coulomb component) has a damping oscillator type behavior.

Its modes of oscillation are "dictated", among others, and by the constant of separation of the variable Λ.

ii) Behaviors at thermal scale resolution

Let us admit that the dynamics of the ablation plasma at thermal scale resolution are "dictated" by its fractal behavior. Then, according to the Theory of Scale Relativity in an arbitrary constant fractal dimension (Mercheş and Agop, 2016), such dynamics are described by the system of equations of fractal hydrodynamics, which in the one-dimensional case takes the form:

$$\partial_t V+V\partial_x V=-\partial_x\left[-2m_0\lambda^2(dt)^{(4/D_F)-2}\times\frac{\partial_x\partial_x(\sqrt{\rho})}{\sqrt{\rho}}\right] \tag{9}$$

$$\partial_t\rho+\partial_x(\rho V)=0 \tag{10}$$

Eq. (9) corresponds to the law of conservation of the specific momentum (momentum of the unit of mass), while Eq. (10) corresponds to the law of conservation of the density of states. In Eqs. (9) and (10) V defines the differential velocity, which is independent of the scale resolution, dt, while ρ defines the density of states, which is dependent on the scale resolution, by means of the fractal velocity V_F,

$$V_F = \lambda(dt)^{(2/D_F)-1}\partial_x \ln\rho \tag{11}$$

with λ the parameter associated with the fractal-non-fractal transition, D_F the fractal dimension of the motion curve and m_0 the rest mass of the fractal fluid entity. Let us also note that the specific fractal potential,

$$Q = -2m_0\lambda^2(dt)^{(4/D_F)-2} \times \frac{\partial_x\partial_x(\sqrt{\rho})}{\sqrt{\rho}} = -V_F^2 - \frac{\lambda(dt)^{(2/D_F)-1}}{2}\partial_x V_F \tag{12}$$

a measure of the non-differentiability, induces the specific fractal force:

$$F_x = -\partial_x Q \tag{13}$$

Next, let us obtain the solution of the system of differential Eqs. (9) and (10) for ablation plasma at thermal scale resolution (thermal component), free of any external constraints. For this we will impose initial and boundary conditions (we will follow the method from Mercheş and Agop, 2016).Thus, we will characterize the initial state of the thermal component both by the discrete value of the velocity,

$$V(x, t = 0) = V_0 \tag{14}$$

as well as by the Gaussian distribution of positions (of parameter α):

$$\rho(x, t = 0) = \frac{1}{\sqrt{\pi}\alpha} e^{-\left(\frac{x}{\alpha}\right)^2} = \rho_0(x) \tag{15}$$

This means that, at time t = 0, the center of the distribution is at $\langle x \rangle_0 = 0$ and has the velocity $\langle V \rangle_0 = 0$. The boundary conditions are given by the relations:

$$\rho(x = +\infty, t) = 0, \quad \rho(x = -\infty, t = 0) = 0 \tag{16}$$

$$V(x = V_0 t, t) = V_0 \tag{17}$$

Because for any given time the average value of the specific fractal force is zero, Eq. (9) can be separated as follows:

$$\partial_x\left(\frac{\partial_x\partial_x\sqrt{\rho}}{\sqrt{\rho}}\right)=\frac{2}{a(t)^2}(x-V_0t) \tag{18}$$

$$\partial_t V+V\partial_x V=\frac{4m_0^2\lambda^2(dt)^{(4/D_F)-2}}{a(t)^2}(x-V_0t) \tag{19}$$

Integrating the Eq. (18) with the boundary conditions (16) implies the solution:

$$\rho(x,t)=\frac{1}{\pi a(t)}e^{-\frac{(x-V_0t)^2}{a(t)}} \tag{20}$$

This function satisfies the initial condition (15) if the initial value of $a(t)$ is of the form:

$$a(t=0)=\alpha^2 \tag{21}$$

The insertion of the Eq. (20) into the continuity Eq. (10) specifies that, for $x=V_0t$, we will have:

$$\frac{1}{2a}\frac{da}{dt}=(\partial_x V)_{x=V_0t} \tag{22}$$

Considering this last result, the differential equation for variable $a(t)$ is obtained by "operating" with Eq. (19). The following results:

$$a\frac{d^2a}{dt^2}+\frac{1}{2}\left(\frac{da}{dt}\right)^2=8m_0\lambda(dt)^{(4/D_F)-2} \tag{23}$$

The solution of the Eq. (23), with the initial condition (21), has the expression:

$$a(t)=\alpha^2+\frac{4\lambda^2(dt)^{(4/D_F)-2}}{\alpha^2}t^2 \tag{24}$$

Now, according to Eqs. (20) and (24), the density of states becomes:

$$\rho(x,t)=$$
$$=\frac{\pi^{-1/2}}{\left[\alpha^2+\frac{4\lambda^2(dt)^{(4/D_F)-2}}{\alpha^2}t^2\right]^{1/2}}exp\left[-\frac{(x-V_0t)^2}{\alpha^2+\frac{4\lambda^2(dt)^{(4/D_F)-2}}{\alpha^2}t^2}\right] \tag{25}$$

Similarly, by integrating the Eq. (19) with the initial condition (14) and the one on the boundary (17), the expression of speed is obtained:

$$V(x,t)=\frac{V_0\alpha^2+\frac{4\lambda^2(dt)^{(4/D_F)-2}}{\alpha^2}tx}{\alpha^2+\frac{4\lambda^2(dt)^{(4/D_F)-2}}{\alpha^2}t^2} \tag{26}$$

Relations (25) and (26) represent the solution of the fractal hydrodynamics equation system for the thermal component of the ablation plasma.

Using these solutions now, we find the expression of the current density of the ablation plasma at the thermal scale resolution (thermal component):

$$J_T(x,t)=\rho(x,t)V(x,t)=$$

$$=\frac{V_0\alpha^2+\frac{4\lambda^2(dt)^{(4/D_F)-2}}{\alpha^2}tx}{\pi^{1/2}\left[\alpha^2+\frac{4\lambda^2(dt)^{(4/D_F)-2}}{\alpha^2}t^2\right]^{3/2}}\,exp\left[-\frac{(x-V_0t)^2}{\alpha^2+\frac{4\lambda^2(dt)^{(4/D_F)-2}}{\alpha^2}t^2}\right] \tag{27}$$

The relationship (27), with constraint:

$$\tau\ll t\ll\frac{V_0\tau^2}{x} \tag{28}$$

where

$$\tau=\frac{\alpha^2}{\lambda(dt)^{(2/D_F)-1}} \tag{29}$$

becomes:

$$J_T(x,t)\to\frac{V_0\alpha^5}{\pi^{1/2}\lambda^3(dt)^{(6/D_F)-3}t^3}exp\left\{-\left[\frac{\alpha}{\lambda(dt)^{(2/D_F)-1}}\right]^2\left(\frac{x}{t}-V_0\right)^2\right\} \tag{30}$$

From here, with notations,

$$x=d,\;\; V_0=\nu,\qquad \left[\frac{\alpha}{\lambda(dt)^{(2/D_F)-1}}\right]=\frac{m_i}{2k_BT} \tag{31}$$

the standard result is obtained (Cremers and Radziemski, 2006):

$$J_T(t)\to\frac{1}{t^3}exp\left\{-\left[\frac{m_i}{2k_BT}\right]\left(\frac{d}{t}-\nu\right)^2\right\} \tag{32}$$

iii) Behaviors at cluster scale resolution

Let us admit that the dynamics of the ablation plasma at the cluster scale resolution are dictated by the diffusion processes. At this scale resolution since the total specific momentum is zero,

$$V = -V_F = -\lambda(dt)^{(2/D_F)-1}\partial_x \ln\rho \tag{33}$$

then the Eqs. (9) and (10) of the fractal hydrodynamic model in the form of the Theory of Scale Relativity in arbitrary constant fractal dimension are reduced to the fractal diffusion equation:

$$\partial_t\rho = \lambda(dt)^{(2/D_F)-1}\partial_x\partial_x\rho \tag{34}$$

The solution of this equation, with proper initial and boundary conditions is of the form:

$$\rho(x,t) = \frac{M}{[4\pi\lambda(dt)^{(2/D_F)-1}\ t]^{1/2}} exp\left[-\frac{(x-x_0)^2}{4\lambda(dt)^{(2/D_F)-1}\ t}\right] \tag{35}$$

where M and x_0 are two integration constants. From here, using the relation (33), the expression of speed is obtained first:

$$V(x,t) = \frac{x-x_0}{2t} \tag{36}$$

then the expression of current density is obtained:

$$\begin{aligned} J_{CL}(x,t) &= \rho(x,t)V(x,t) = \\ &= \frac{M(x-x_0)}{[16\pi\lambda(dt)^{(2/D_F)-1}]^{\frac{1}{2}}t^{\frac{3}{2}}} exp\left[-\frac{(x-x_0)^2}{4\lambda(dt)^{(2/D_F)-1}\ t}\right] \end{aligned} \tag{37}$$

In the notations:

$$D = \lambda(dt)^{(2/D_F)-1}\ , \quad x \equiv d,\ \ x_0 = 0 \tag{38}$$

and with the meanings from (Nottale, 2011) the standard result is obtained (Cremers and Radziemski, 2006):

$$J_{CL}(t) = \frac{M}{[16\pi Dt\ ]^{1/2}}\left(\frac{d}{t}\right) exp\left[-\frac{d}{4D}\left(\frac{d}{t}\right)\right] \tag{39}$$

iv) Behaviors at global scale resolution

Assuming the functionality of the principle of superposition of the scales resolutions, the current density at the global scale resolution will be expressed as a sum of the current density at the Coulomb scale resolution, given by the relation (7), of the current density at the thermal scale resolution, given by the relation (32) and of the current density at the cluster scale resolution, given by the relation (38). The expression results:

$$J_G = Aexp(-\delta t)\sin(\Omega t + \varphi) + Bt^{-3} exp\left[-\left[\frac{m_i}{2k_B T}\right]\left(\frac{d}{t} - v\right)^2\right] + \\ + Ct^{-1/2}\left(\frac{d}{t}\right) exp\left[-\frac{d}{4D}\left(\frac{d}{t}\right)\right] \tag{40}$$

where A, B and C are constant.

3. Conclusions

In our opinion, the three “behaviors” can describe the diversity of hydroxyapatite ablation plasma dynamics, which can be seen from ultra-fast imaging records. We mention that the “analyzes” of optical spectroscopy highlight the presence of the first two behaviors (Coulomb type and thermal type behavior). The third “behavior”, that is, the cluster type, requires, in order to be highlighted, completely special “techniques” (for example, shadowgraphy, infrared absorption, etc.), but we did not have these techniques used, considering that “imaging” is sufficient for the purpose pursued by us.

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MODELAREA TEORETICĂ A DINAMICILOR
UNEI PLASME DE ABLAŢIE

(Rezumat)

Utilizând Teoria Fractală a Mişcării sub forma Relativităţii de Scară în dimensiunea fractală arbitrară şi constantă, sunt analizate comportamente variate ale unei plasme de ablaţie. Mai precis, se arată că expansiunea plasmei de ablaţie implică trei „momente” distincte: „momentul” Coulomb, „momentul termic” şi „momentul cluster”.